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## Use of Navier-Stokes Analysis in Section Design

by

Phuc N. Nguyen

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## NOMENCLATURE

$\alpha$	Angle of attack
$C$	Chord
$C_L$	Lift coefficient
$C_p$	Pressure coefficient
$\epsilon$	Turbulent dissipation
$f/f_m$	Chordwise distribution of camber as normalized by the maximum value
$f_m/C$	Maximum camber normalized by chord
$k$	Turbulent kinetic energy
$t/t_m$	Chordwise distribution of thickness as normalized by the maximum value
$t_m/C$	Maximum thickness normalized by chord
$U_o$	Freestream velocity
$u$	Fluctuating streamwise velocity
$v$	Fluctuating transverse velocity
$x$	Streamwise distance
$y$	Transverse distance

## ABSTRACT

*The Navier-Stokes analysis method and a design technique based on conformal mapping are combined to develop new 2-D thick sections. The Eppler-Somers design technique allows for fast design of arbitrary section shape. The well-validated David Taylor Navier-Stokes code is used to optimize the thickness of the section. From previous experimental results, the turbulence characteristics in the near-wake region correlate with the pressure spectra on the trailing edge of a 2-D lifting surface. Therefore, the turbulent kinetic energy, and the Reynolds shear stress are used as design parameters to develop new 2-D sections. Minimizing these parameters is assumed to provide desirable boundary-layer and wake characteristics. The characteristics of one new section are compared with those of a baseline section to demonstrate the new foil design method.*

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## ADMINISTRATIVE INFORMATION

This project was performed by the Propulsor Technology Branch (Code 1544) of the David Taylor Research Center (DTRC) under Project Number 1-1506-060-34 for the Office of Naval Technology.

## INTRODUCTION

In this report, with the aid of a Reynolds-averaged Navier-Stokes (N-S) analysis, the potential of tailored blade sections instead of standard NACA sections is explored for optimization of propeller performance. Propeller designers normally use sections with NACA 16 or NACA 66 thickness distributions and an  $a=0.8$  meanline. Due to recent improvements in computational capability, it is now feasible to shape the section to achieve a specific design goal, whether it be maximizing efficiency, minimizing cavitation, or boundary-layer control. Also, for some applications, it is desirable to maximize the section thickness without degrading the propeller performance by massive flow separation. The motivation for this new section work is due to the experimental results of Gershfeld et al. [1], and Huang et al. [2], which have shown that the pressure spectra on the trailing edge are related to the turbulent flow characteristics in the near-wake region. The turbulent flow data in Ref. [2] were used for validation of the N-S analysis [3]. The validation showed that the David Taylor Navier-Stokes DTNS code [4] can be useful in new section design.

The preliminary design is based on the Eppler-Somers conformal mapping technique [5]. This technique allows for fast design of arbitrary section shapes with minimum effort from the designer. Originally developed for wing section design in incompressible flows, the code can calculate the boundary-layer development with an integral method. The

code handles high Reynolds number flows with an empirical turbulent transition model. At DTRC, the Eppler-Somers code has been mainly used for cavitation work [6]. Its analysis capability with a panel method provides quick calculation of the cavitation bucket for a given section shape. In this report, the N-S analysis is employed where the turbulence characteristics are needed. The differential boundary-layer method could have been used, but Ref. [2] showed limitations of the method in the turbulent wake calculations when the mean velocity profiles and the Reynolds shear stress profiles need shifting on the order of one wake thickness to match the experimental data. The following section describes how the new section is designed.

## OUTLINE OF ANALYTICAL METHOD

The objective is to maximize the thickness of a 2-D lifting foil and to reduce trailing edge turbulent kinetic energy without incurring significant flow separation. This is achieved by control of the turbulent boundary-layer and wake characteristics through careful shaping of the 2-D section. The N-S analysis is used in the last stage of a foil design process to calculate the turbulent flow characteristics and to provide feedback for thickness adjustment. The flow chart in Fig. 1 summarizes this iterative design process. Each step in the design procedure is now described in detail.

## THIN SECTION DESIGN WITH POTENTIAL FLOW METHOD

The Eppler-Somers code was chosen for its fast and accurate capability to produce 2-D sections. The features of the code are: conformal mapping method for design of sections with prescribed flow characteristics, panel method for inviscid flow analysis of the designed section over a range of angles of attack, and an integral boundary-layer method for preliminary viscous analysis. In this code, the desired pressure distribution at design angle of attack is not actually entered directly as input. This would be single-point design. The Eppler-Somers formulation can do multi-point design. This is achieved by dividing the foil section into segments. Each segment can satisfy a design requirement, which is a desired velocity distribution at a desired angle of attack. This multi-point design capability is possible because the function that relates the velocity distribution to the angle of attack varies only with the position on the section surface [5]. Iteration is necessary to achieve the requirements of a multi-point design, whether it be widening the cavitation bucket, or the laminar flow bucket.

In the Eppler-Somers code, the inputs are 5 angles of attack for 5 segments around the section surface. Another input parameter is the chordwise location at which the pressure recovery starts. The further upstream the pressure recovery starts, the more gradual the pressure gradient, and the less likely the flow is to separate. On the other hand, starting the pressure recovery too early means that the hydrofoil does not develop as much lift as it theoretically could. In order to achieve the same lift, the "early-recovery" hydrofoil will have lower minimum  $C_p$  than that of the "late-recovery"

hydrofoil. For standard propeller sections, this position is at 80% chord, corresponding to the so-called  $a=0.8$  meanline shape.

To illustrate with the design of a new section, the 5 pairs of angles and the corresponding locations are specified such that the pressure distribution on the pressure side is relatively flat, and that on the suction side has a strong flow-accelerating region from the leading edge to the minimum pressure point at mid-chord. The values of these parameters are presented in the following table.

Table 1. Input values for the thin-foil design of the new section.

Positions on the transformed circle (deg)	170.	0.	192.	219.	360.
Corresponding angles of attack (deg)	2.09	1.72	0.0	0.69	2.8
Position for pressure recovery (deg)	90.0				

Note that the section designed at this stage is thin ( $< 10\% t/C$ ). The code does not iterate for boundary-layer displacement effect; therefore, it can not handle large thicknesses well. The resulting pressure distribution and the section itself for this stage are shown in Fig. 2. It has the desired features as specified earlier. At this point, the section is split into the thickness distribution and the camber distribution as shown in Fig. 3.

From the given camber distribution, the ideal angle of attack  $\alpha_i$  can be calculated for any camber/chord ratio using the conventional thin-foil theory [7] which assumes that the mean camber line represents the section. Therefore, the " $C_L$  vs.  $\alpha$ " linear function can be calculated since the slope is  $2\pi$ . The thin-foil theory is used in this design method to estimate the camber ratio and the  $\alpha_i$  for a given  $C_L$ . Fig. 4 shows the  $C_L$  vs.  $f_m/C$  for the new section, and the corresponding  $C_L$  vs.  $\alpha_i$ . The thickness effect is then added, and the resulting section analyzed by the Reynolds-averaged N-S method. From viscous analysis, it is seen that the lift coefficient goes down as the thickness is increased. The trend is reverse for an inviscid analysis. Therefore, it is not safe to design thick ( $> 15\% t/C$ ) lifting surfaces using inviscid tools alone.

## THICK SECTION DESIGN WITH NAVIER-STOKES METHOD

Given the basic thin section, the N-S analysis is used iteratively to obtain the final design. Two parameters are used in the iteration with the N-S analysis: thickness, and

angle of attack. The criterion for the final design is "no-separation" in the range of  $\pm 4^\circ$  around the design angle of attack with the maximum thickness as large as possible. The  $\pm 4^\circ$  range is normally the fluctuation of angle of attack that a propeller section sees in straight-ahead operation.

There are two steps in obtaining a N-S solution: 1) geometry preparation, including grid generation, and 2) flow calculation. For this study, the grid generation is based on the work of Coleman [8], which uses partial differential equations to define a body-fitted grid. Here, a multi-zone grid is used for better control of the grid structure, which is especially useful when combined with a multi-zone flow code such as the DTNS code [4]. For a high Reynolds number flow (greater than  $10^6$ ) the first grid point should be as close as  $10^{-5}$  chord length away from the body, so that the sub-layer can be resolved. Fig. 5 shows a 3-zone grid used for computing the flow over a foil with Reynolds number of  $5 \times 10^6$ . The flow solution procedure is described in Ref. [3]. It is summarized in the following paragraphs.

Artificial compressibility [9] is used in solving the 2-D incompressible N-S equations. The N-S and the  $k - \epsilon$  equations [10] contain both first derivative convective terms and second derivative viscous terms. The viscous terms are numerically well-behaved terms and central differencing is used. An upwind differenced Total Variational Diminishing (TVD) scheme [11] is used for the convective part of the equations. The equations are solved in an implicit coupled manner using approximate factorization. This creates a diagonally dominant system which requires the inversion of block tri-diagonal matrices. An important quality of any scheme is its convergence rate. The diagonal dominance of the present method allows large time steps to be used for fast convergence.

The solution starts with initial estimates for the kinetic energy and dissipation fields. Here, the flow field calculated with the Baldwin-Lomax [12] turbulence model is supplied to the  $k - \epsilon$  model equations to get the estimates. The N-S equations are solved to the wall with proper no-slip boundary conditions for all cases. With the Baldwin-Lomax turbulence model, the Van Driest mixing-length model takes care of the near-wall region. The N-S equations and the  $k - \epsilon$  model equations are iterated in pseudo-time until convergence is obtained.

## DISCUSSION OF PERFORMANCE PREDICTIONS

The experimental data [2] for two 2-dimensional lifting foils are used as bench-marks for the N-S analysis. Both foils have an  $a=0.8$  meanline and NACA 16 Type II thickness distribution. The validation of the N-S code is documented in Ref. [3], which concluded that the DTNS code can provide reasonable predictions of the viscous flow performance.

## MEAN FLOW CHARACTERISTICS

The baseline section, shown in Fig. 6, has an  $a=0.8$  meanline and a NACA 16 thickness distribution with maximum thickness of 17.16% chord and camber of 4.79%



chord. The design lift coefficient is approximately 0.69. The chosen Reynolds number is  $5 \times 10^6$  to match the conditions for 1/4-scale tests of naval propellers. Fig. 6 also shows the difference between the pressure distribution calculated by N-S method and by panel method [13]. The angle of attack in Fig. 6 is  $4^\circ$ ; the design angle is only  $1.1^\circ$ . Due to the effect of the thick boundary-layer and flow separation, the angle of attack has to be high to achieve the design lift. For the same operating conditions, the velocity vector plot in Fig. 7 shows the flow separation of the baseline section on the suction side at approximately 88% chord. This flow separation will correlate with high turbulence activity according to the findings in Ref. [2].

The final design of the new section is produced, after some N-S iterations with different thicknesses and angles of attack. The geometry is shown in Fig. 8 with the pressure distribution at design angle,  $2.8^\circ$ . The maximum camber and thickness ratios are 5.41% chord and 14.73% chord, respectively. The lift coefficient from this pressure distribution is approximately 0.69, the same as the design value. The desirable characteristics for the boundary-layer development are presented in this pressure distribution. Maximum suction peak is around 50% chord on the suction side; steep pressure recovery follows immediately, then the gradient becomes milder to minimize flow separation as the trailing edge is approached. Also, the pressure side pressure distribution is rather flat over most of the surface; this should reduce the turbulent kinetic energy. The velocity vector plot in Fig. 9 shows attached flow on both the pressure and suction side at design angle of attack. Fig. 10 shows the  $C_L$  vs.  $\alpha$  performance of the new section. Note that there is no lift degradation at high angles. Also, the results of the thickness parametric study show that the new section maintains considerable lift as thickness is increased up to 22% chord, as compared to the standard NACA-16 section (see Fig. 11). The Reynolds number for this parametric study is  $2 \times 10^6$  which simulates the average conditions for propeller tests in the 36-inch Variable Pressure Water Tunnel at DTRC. It should be pointed out, however, that the weak point of this design is the low minimum  $C_p$  compared to that of the baseline.

## TURBULENCE CHARACTERISTICS

The attached flow field of the new section produces lower turbulent kinetic energy as seen in Fig. 12, and lower Reynolds shear stress as seen in Fig. 13 when compared to the baseline section. This trend is more pronounced for the suction side than the pressure side, indicating flow separation on the suction side. And the pressure side of the new section has low turbulence activity because the pressure distribution there is almost flat over the entire surface. Refs. [1,2] showed correlation of turbulence characteristics in the near-wake region with the pressure spectra at the trailing edge. How this correlation function behaves is not well understood. According to Ref. [1], mild flow separation on the suction side produces higher pressure on the trailing edge at low frequency. But the high frequency pressure fluctuations are reduced. The goal of this project is to minimize flow separation and the accompanying high pressure fluctuations on the

trailing edge for a given loading. Figs. 12 and 13 show that the new section achieves this. The trailing edge, however, should be thickened and beveled when constructed for experimental evaluation.

## CONCLUSIONS AND RECOMMENDATIONS

In this report, a design method for 2-D hydrofoil sections is developed by combining a conformal mapping technique with an iterative N-S analysis. The key point of this method is that thickness is considered an important part of the design process for thick hydrofoils. Results show that a new section designed by this method has lower near-wake turbulence activity, for the same lift coefficient, than the baseline. And for the same thickness, the new section carries more lift than the baseline section. This trend is more important when the Reynolds number is decreased as in model-scale experiments. Since the design goal is to maximize thickness with minimum flow separation, this new section is not recommended for other applications in which high thickness is not needed. This particular new section will certainly have poor cavitation performance. Nevertheless, this report illustrates that the Reynolds-averaged N-S analysis is very useful in guiding 2-D section design. The N-S analysis, however, can only give insight about the magnitude and the spatial distribution of the mean flow and the Reynolds stresses. The spectral behavior is entirely unknown.

Until better turbulence models, numerical techniques, and computers become more accessible, the N-S analysis should only be used for final design fine-tuning or off-design predictions as was done in this case. From the results, further work is recommended to: (1) develop a new section with thicker trailing edge; 2) develop a series of new sections with different locations of the minimum pressure point on the suction side and experimentally evaluate them in the same manner as in Refs. [1,2]; and 3) concentrate on the development of turbulence models that can calculate more accurately turbulent flows with strong adverse pressure gradient, and even separation.

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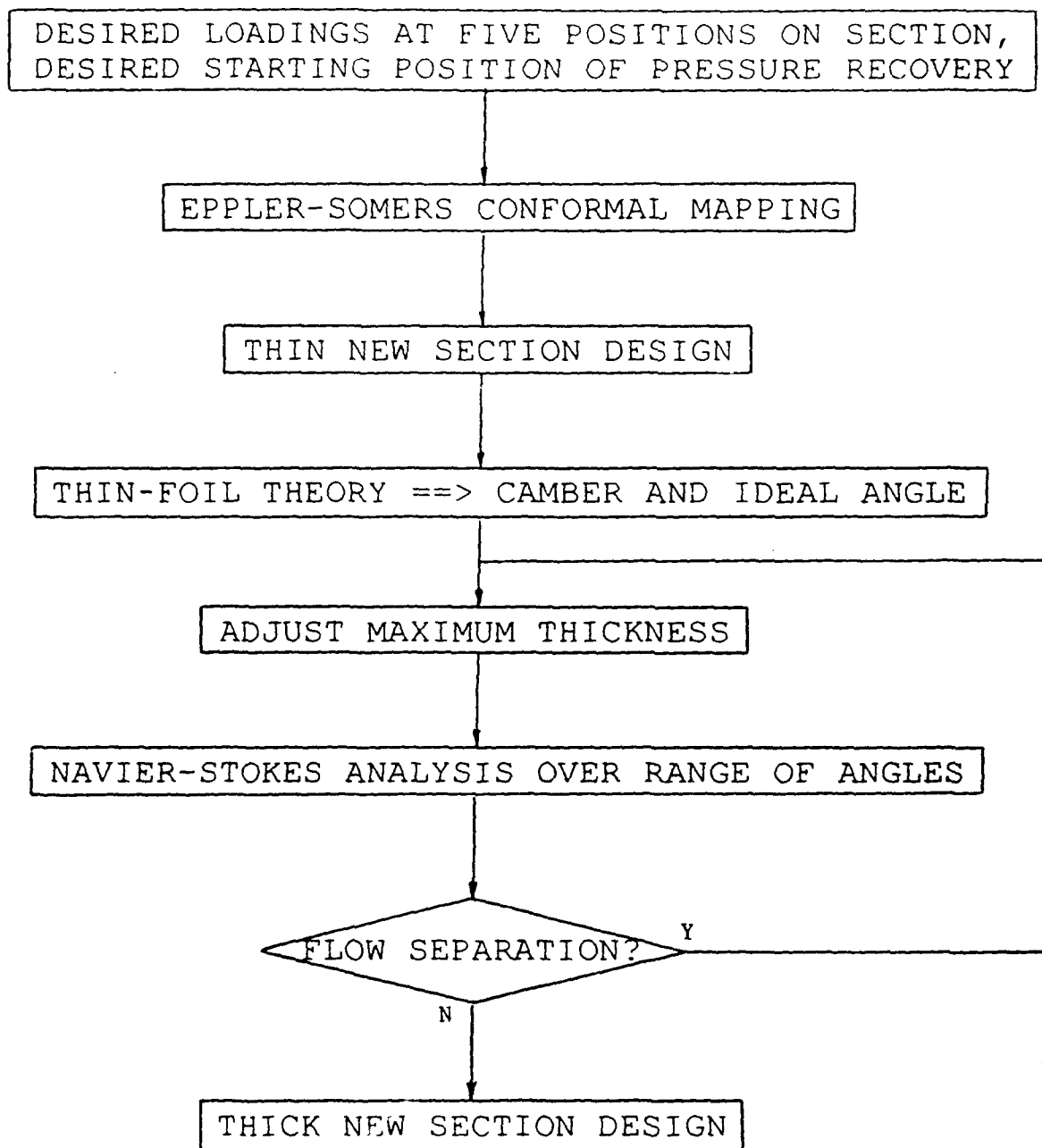


Fig. 1. Flow chart of the new section design procedure.

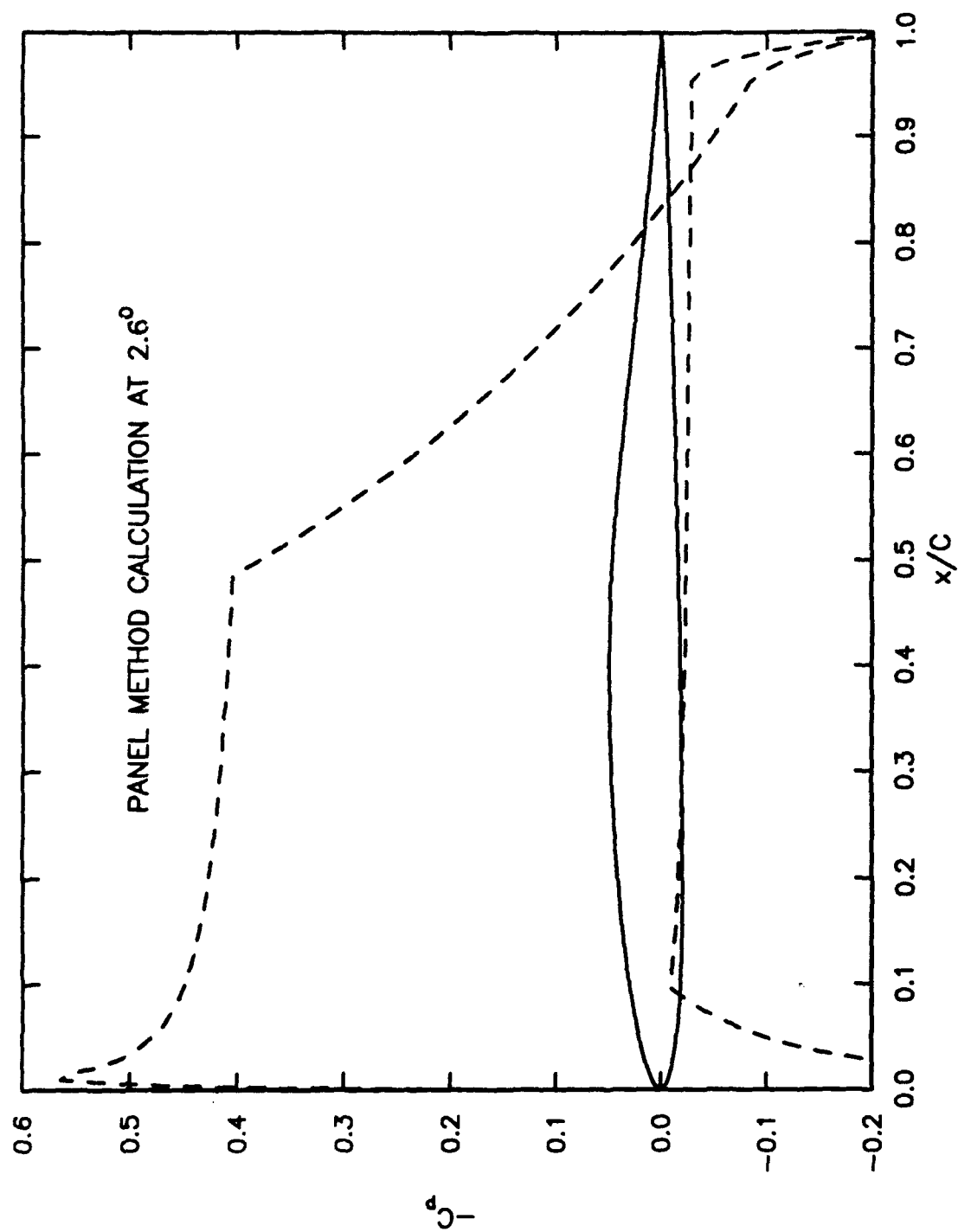


Fig. 2. Pressure distribution of the thin-foil design for the new section.

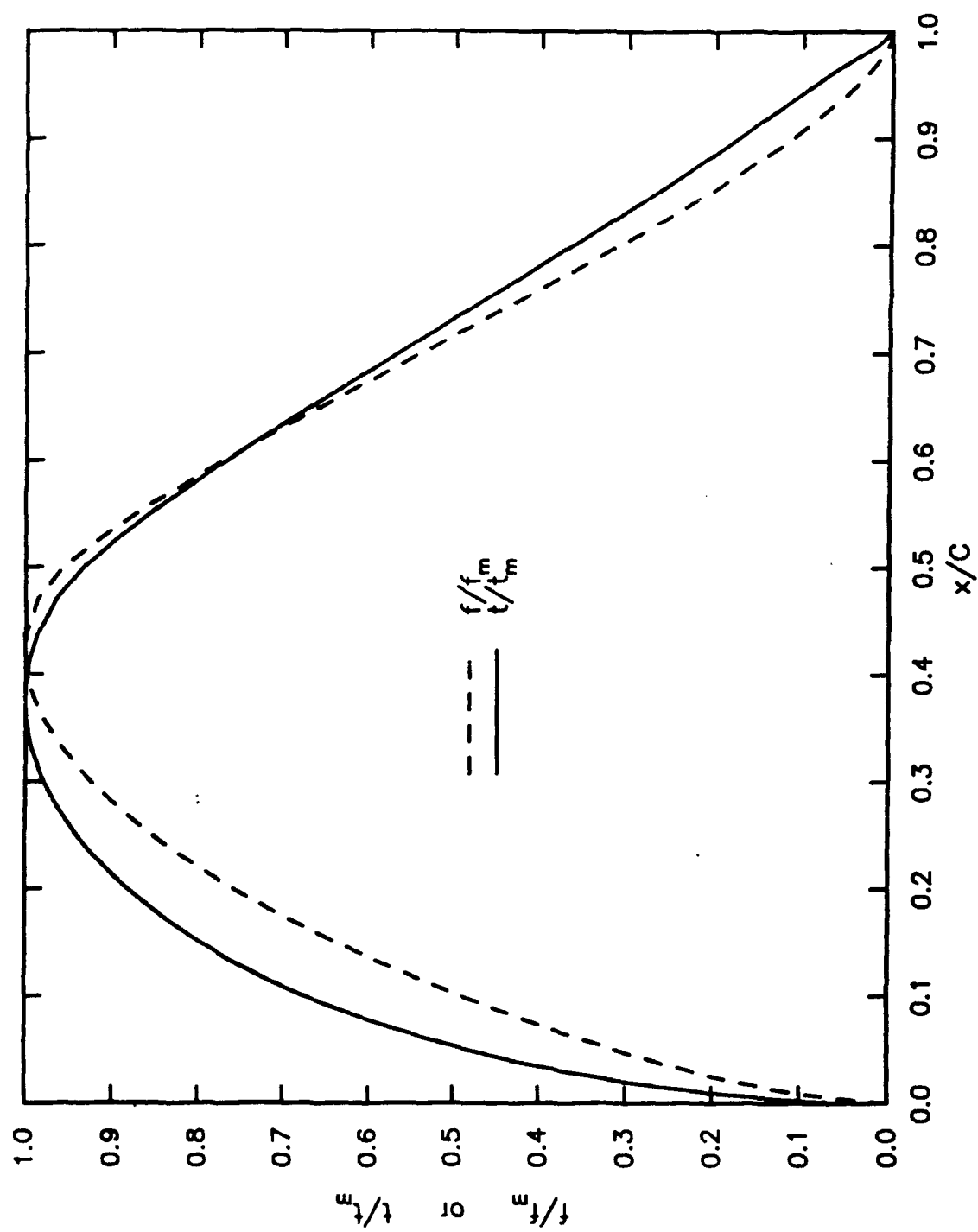


Fig. 3. Camber and thickness distributions of the new section.

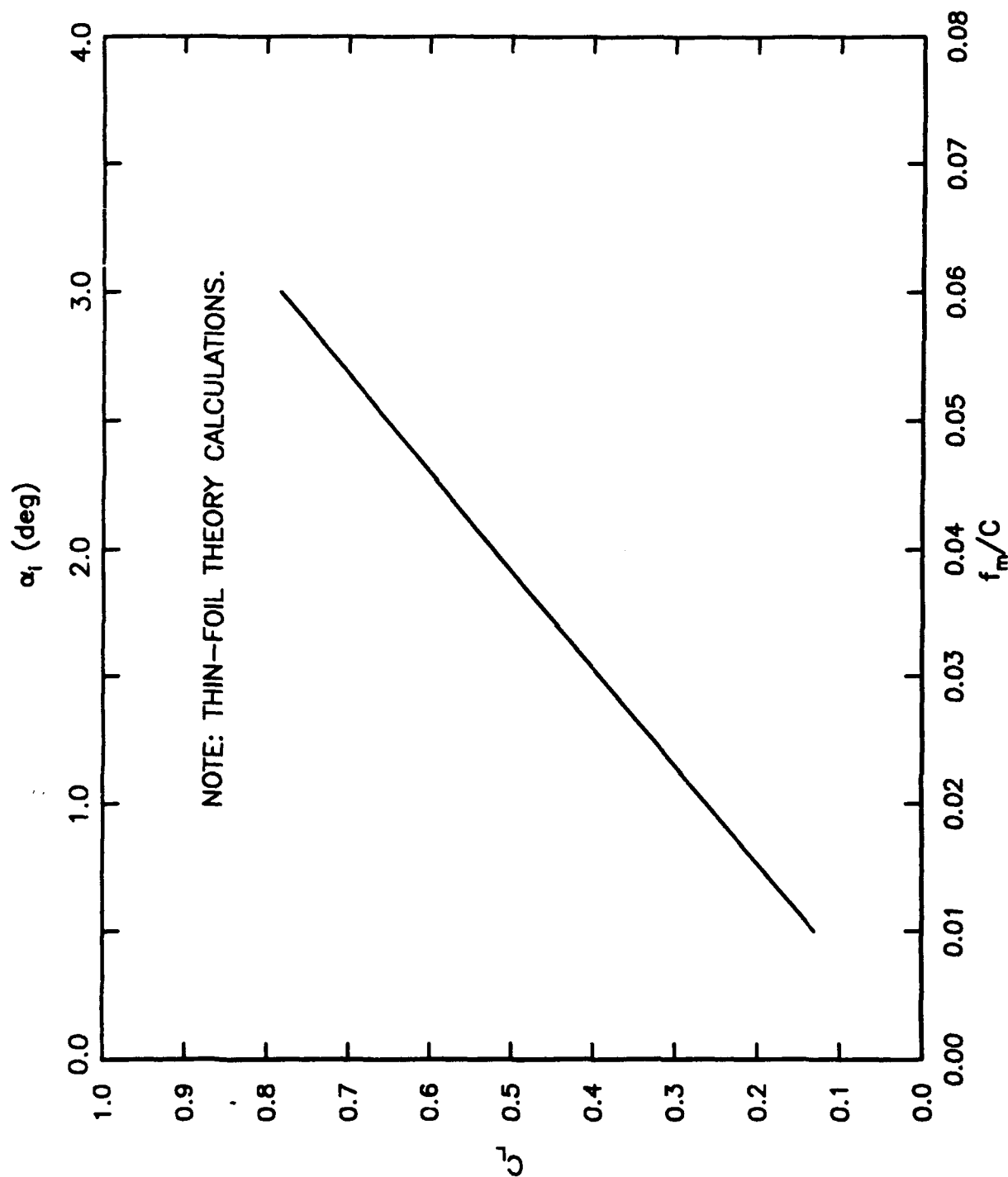


Fig. 4. Lift coefficient vs. camber ratio or ideal angle of attack for the new section.

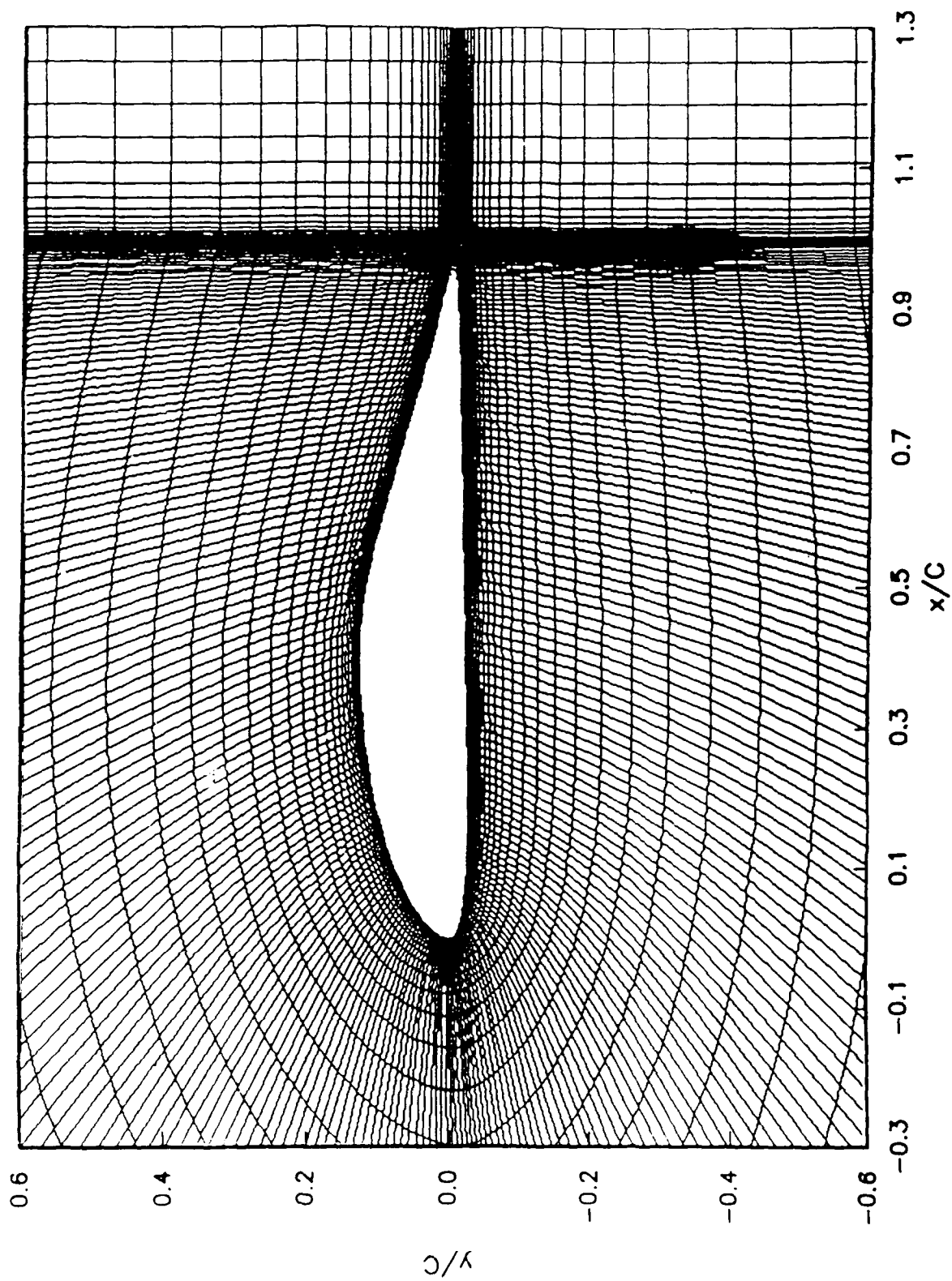


Fig. 5. Three-zone C-type grid for Navier-Stokes calculations.

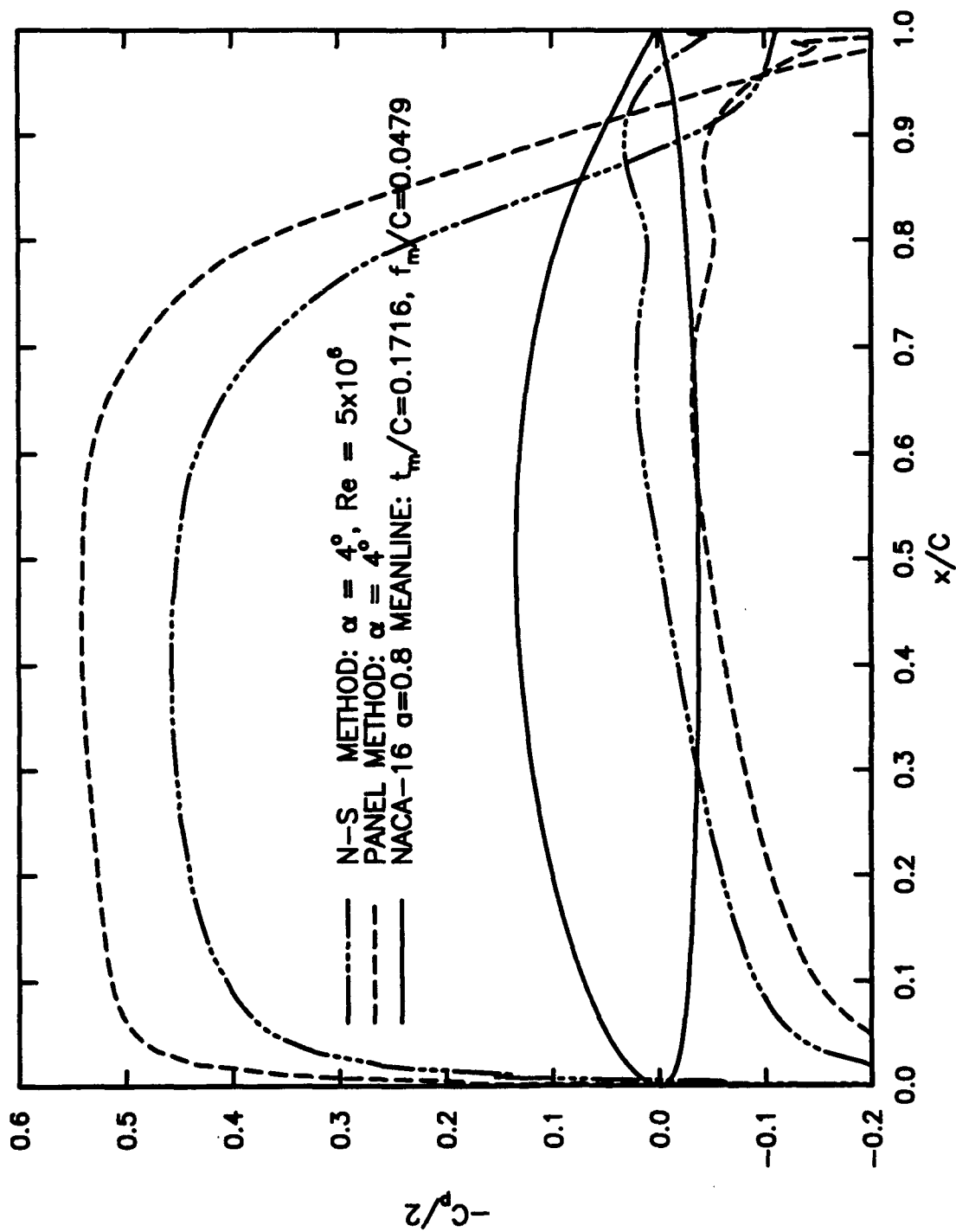


Fig. 6. Geometry and pressure distribution of the baseline section.



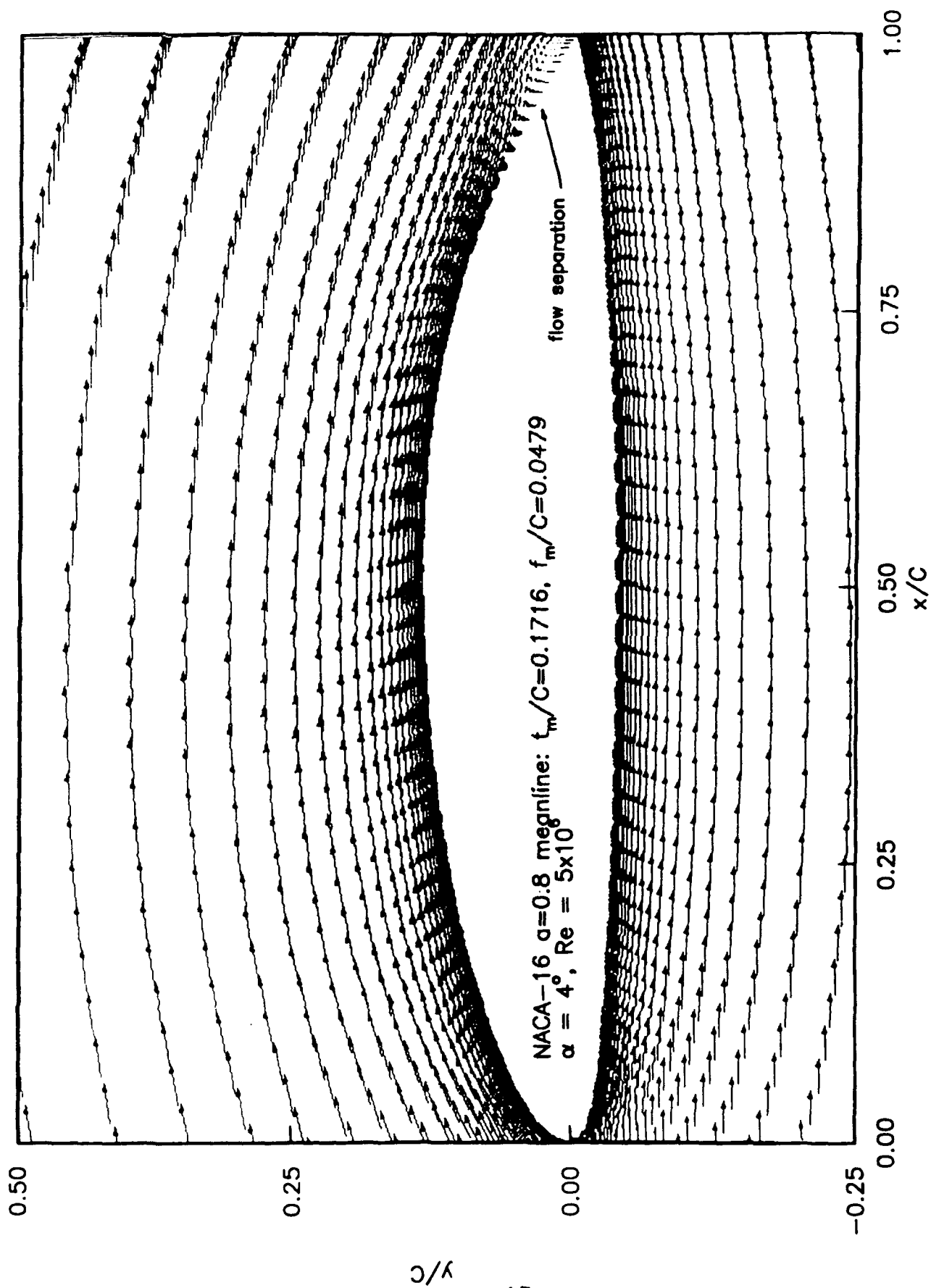


Fig. 7. Velocity field around the baseline section.

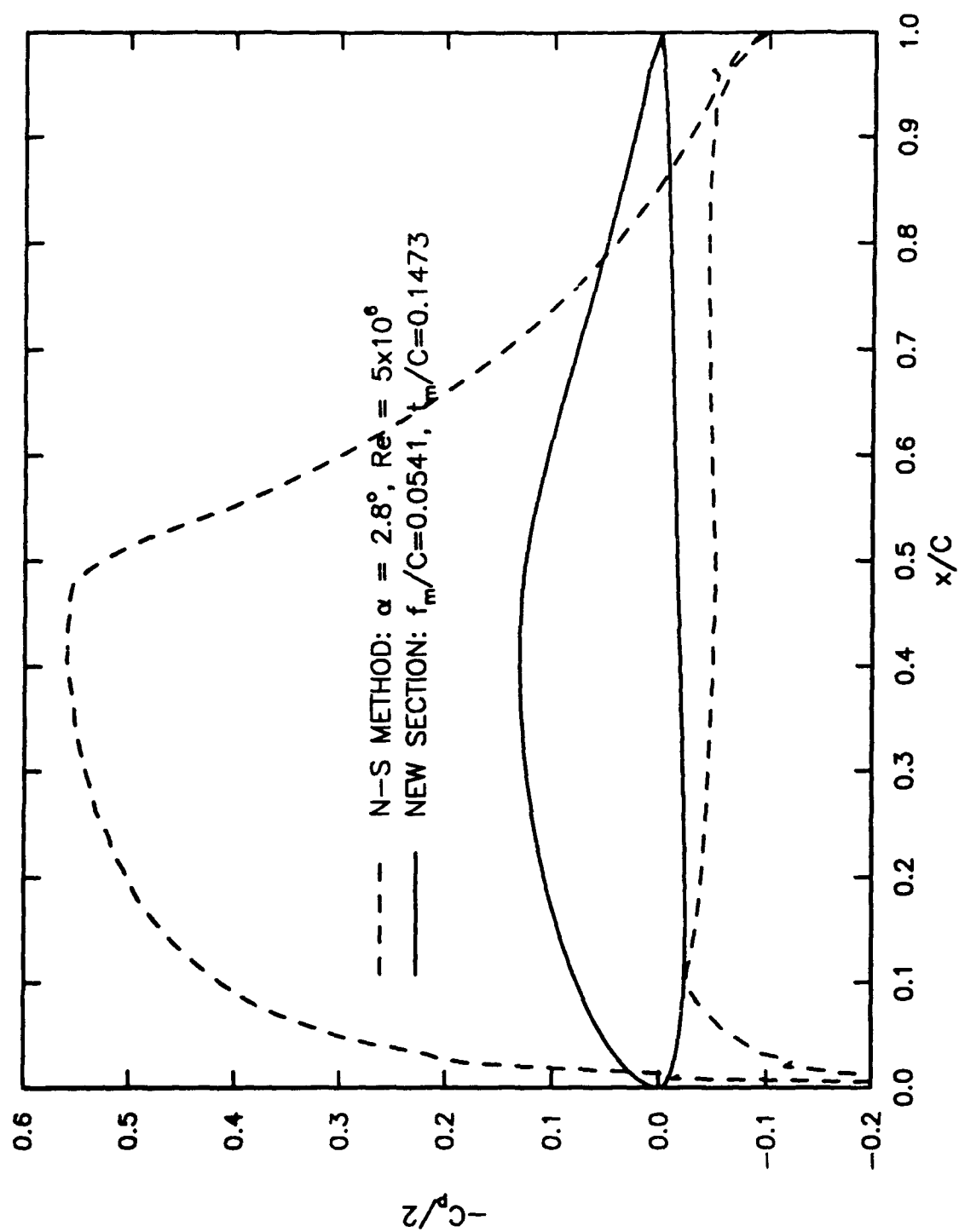


Fig. 8. Geometry and pressure distribution of the new section.

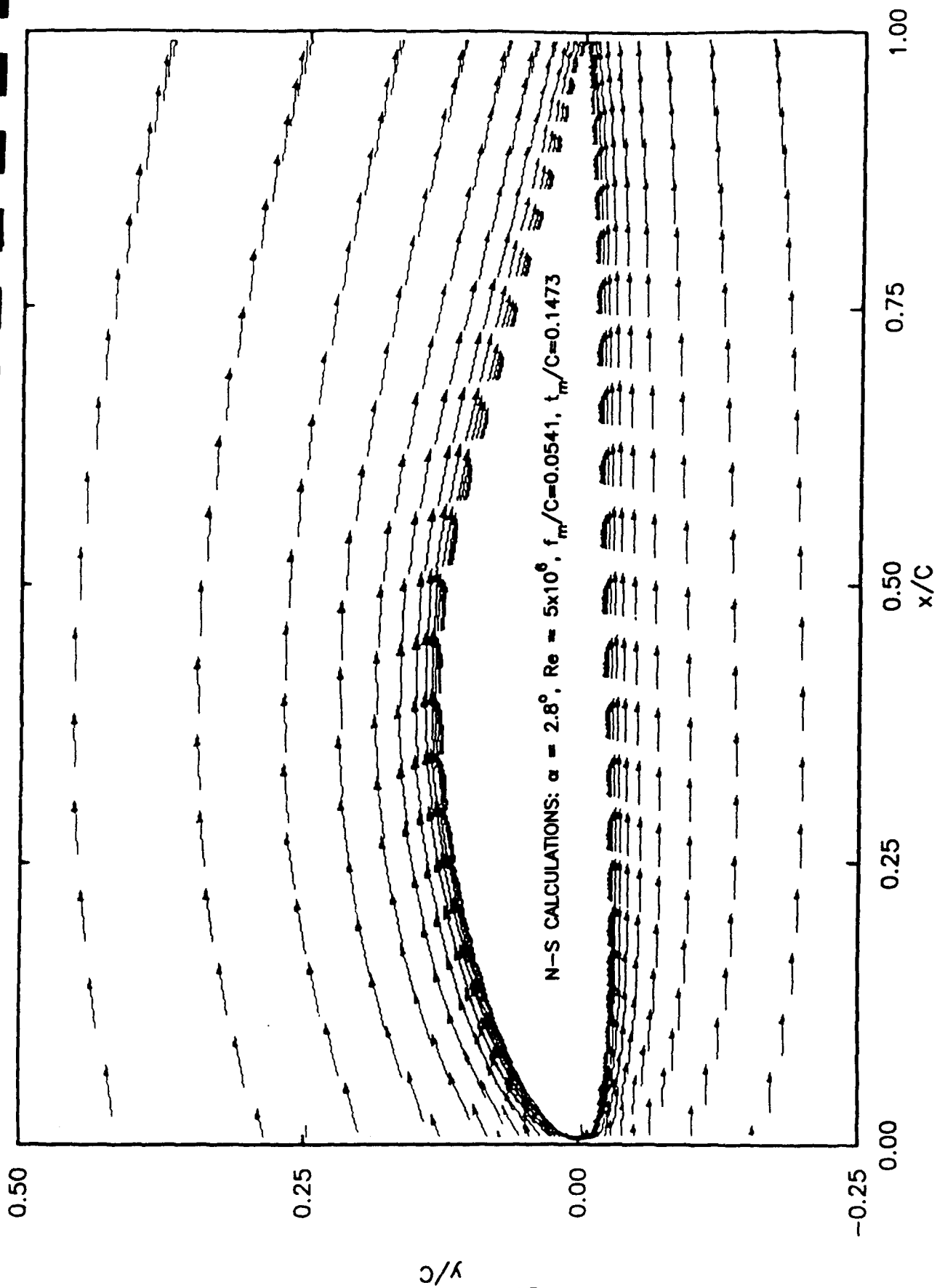


Fig. 9. Velocity field around the new section.

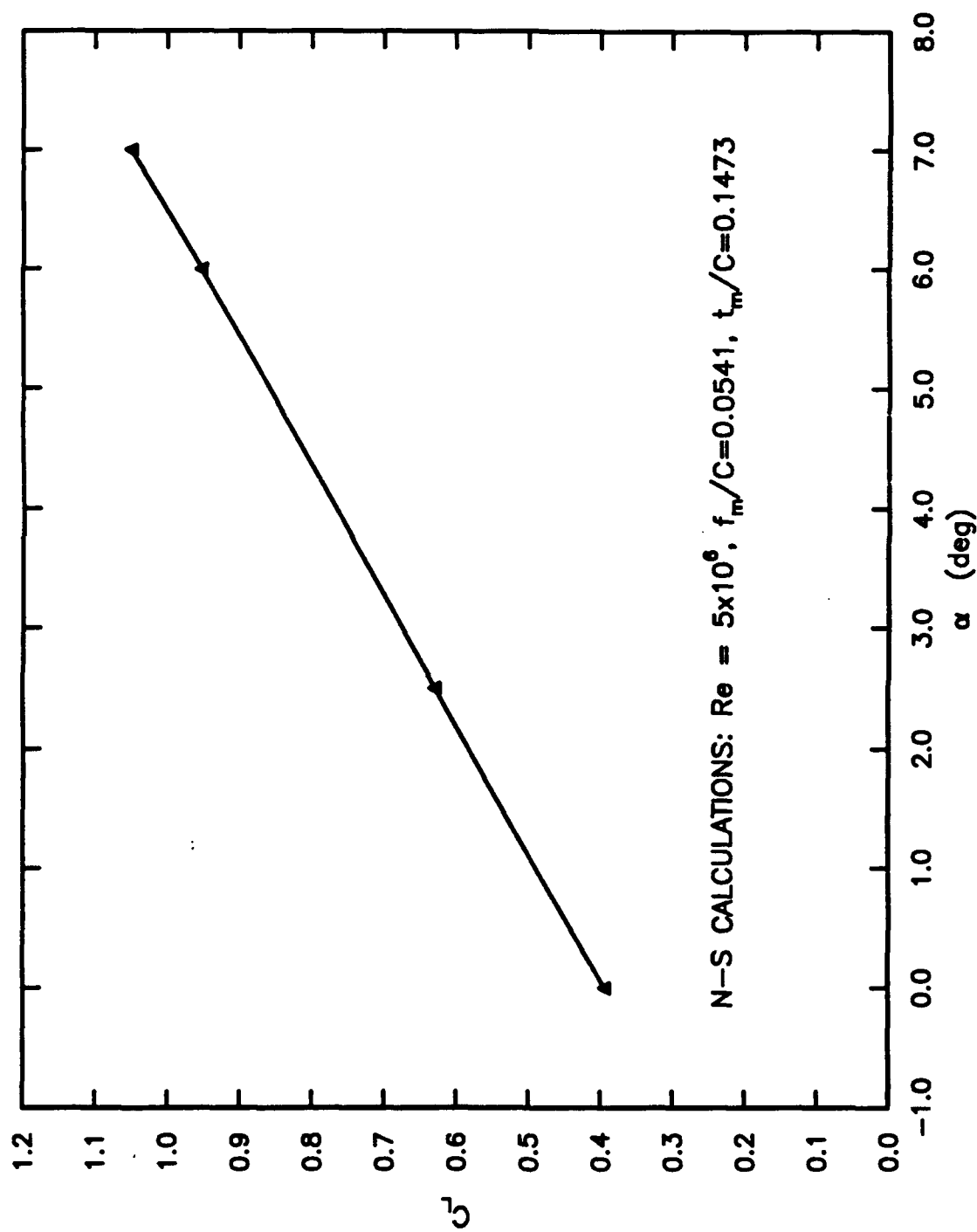


Fig. 10. Lift coefficient vs. angle of attack for the new section.

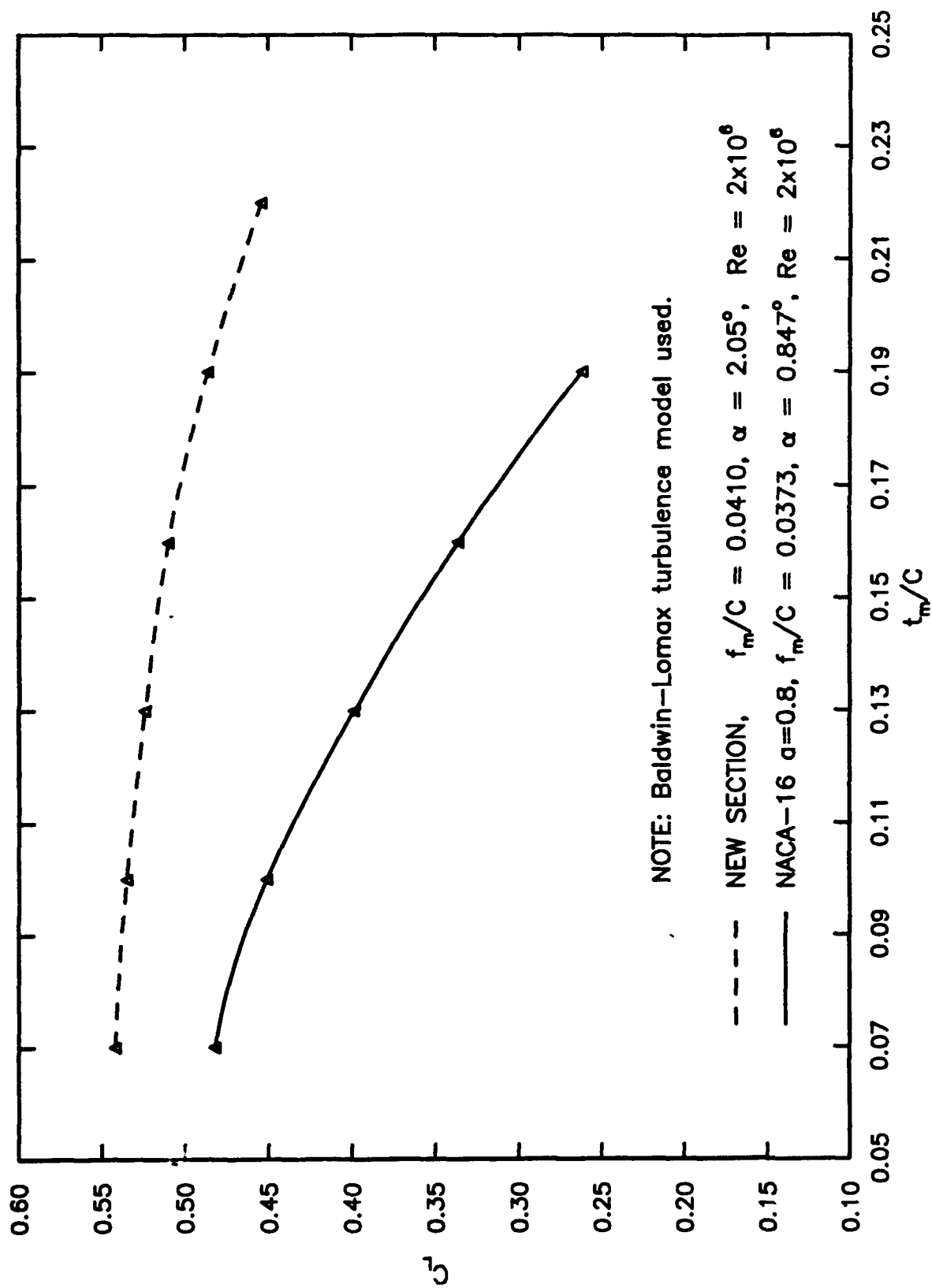


Fig. 11. Lift coefficient vs. thickness for the new, and the baseline section.

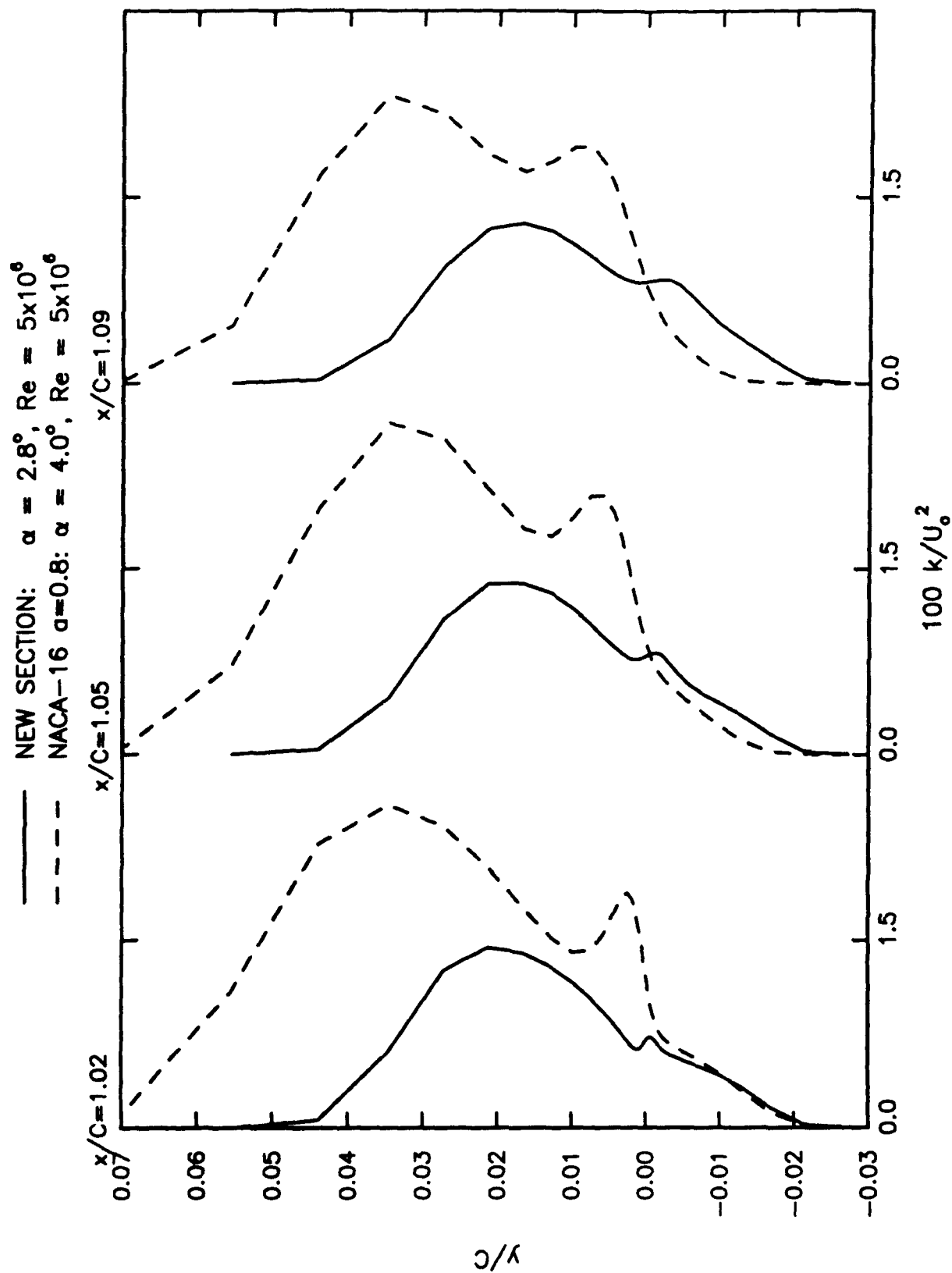


Fig. 12. Turbulent kinetic energy profiles for the new, and the baseline section.

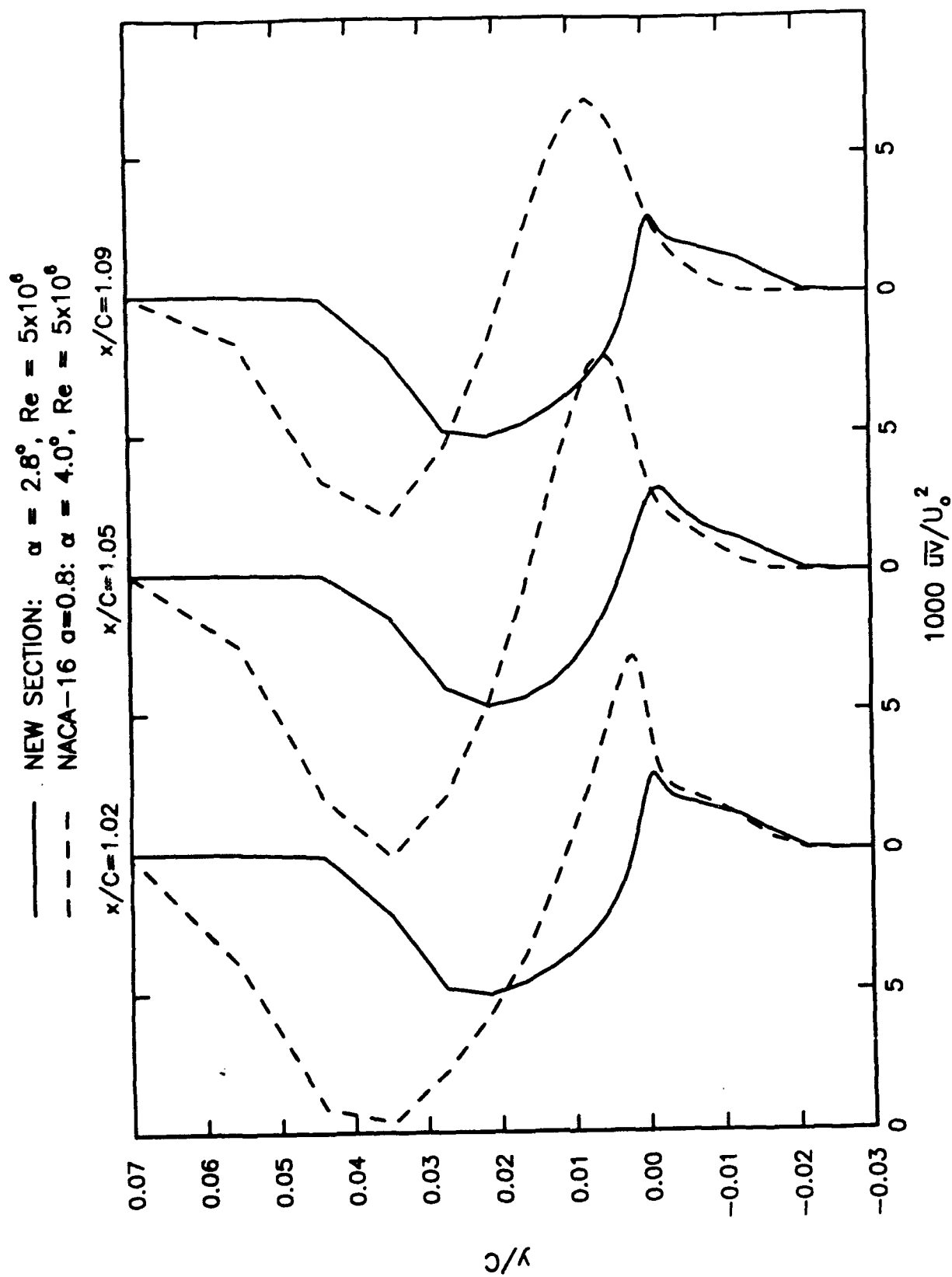


Fig. 13. Reynolds shear stress profiles for the new, and the baseline section.

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